УДК 519.615.7+621.384.634.5

NUMERICAL OPTIMIZATION OF ACTIVELY SCREENED SC MAGNETS COIL GEOMETRIES

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This work is concerned with numerical optimization of actively screened SC magnets coil. An additional SC coil is suggested to shield stray fields. This SC coil only slightly increases the superconductor volume but drastically reduces the total magnet weight and its cost, and permits one to avoid such unpleasant features of the cold yoke as an additional energy losses due to steel magnetization in a pulse mode and nonlinearities with steel saturation at high fields. We consider analytically the field of a shielded magnet and show how the volume of superconductor should be increased to get a complete active shielding. The design of SC coil geometries is performed by solving the optimization problem. Some results of actively screened dipole coil optimization are presented.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Численное моделирование обмотки активно экранированных СП-магнитов

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Проводится численное моделирование активно экранированных СП-магнитов. Для экранировки поля вне магнита используется дополнительная СП-обмотка. Данная обмотка позволяет существенно уменьшить вес магнита и его стоимость и избежать потерь энергии и нелинейностей в магнитном поле вследствие насыщения ферромагнетика при сильных полях. Аналитически исследуется возможность полной экранировки магнитного поля с помощью введения дополнительной обмотки. Реальная геометрия обмотки получена путем решения задачи условной оптимизации. Представлены результаты моделирования.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

1. Introduction*

In modern SC magnets the field strength and its quality are determined by a SC coil and its shape. Nevertheless, a cold steel yoke constitutes 80% and more of the magnet weight which significantly increases the magnet cost and time of its cool-down and warm-

*This work was supported by the RFBR under Grant No.95-01-01467a

up [1]. The steel yoke increases the magnet field only by 20—25%, and its main role is to shield stray fields. This task can be successfully resolved by an additional SC coil that only slightly increases the superconductor volume, but drastically reduces the total magnet weight and its cost, and permits one to avoid such unpleasant features of the cold yoke as an additional energy losses due to steel magnetization in a pulse mode and nonlinearities with steel saturation at high fields.

We consider analytically the field of a shielded magnet and show how the volume of superconductor should be increased to get a complete active shielding. The design of SC coil geometries requires solving the sophisticated optimization problem [2]. We present some results of actively screened dipole coil optimization.

2. Ideal Screened Fields

Dipole and quadrupole fields required in high energy accelerators are produced by SC coils of two shapes [3]: overlapping ellipses and cos θ for a dipole field and crossed ellipses or cos 2θ for a quadrupole one which provide ideal fields. Let us consider an elliptical conductor with axes a, b and a current density j immersed into a bigger or screening one with axes a_s, b_s and a current density j_s. The magnetic field outside this pair of conductors is equal to [4]

$$B_{\text{out}} = \mu_0 \left(\frac{jab}{z + \sqrt{z^2 - c^2}} + \frac{j_s a_s b_s}{z + \sqrt{z^2 - c_s^2}} \right).$$

This equation shows that the outside field is zero if the following two conditions are fulfilled

$$jab = -j_s a_s b_s , \tag{2.1}$$

$$c = c_s . \tag{2.2}$$

Eq. (2.2) means that the total currents in both conductors are the same but of opposite directions, and from (2.3) it follows that the focuses of both elliptical conductors coincide. Now suppose that

$$a_s = ka \quad \text{and} \quad k > 1.$$

Then according to Eq. (2.2) and (2.3), we find

$$j_s = -j\frac{b}{k\sqrt{b^2+(k^2-1)a^2}} \approx -\frac{j}{k^2}. \tag{2.3}$$

The last approximation is valid if $a \approx b$ and $k >> 1$. This pair of screened conductors is used below to construct ideal screened fields.

If we now take another pair of similar conductors with as opposite currents as before and overlap (or cross) them, we shall get an ideal dipole (quadrupole) magnet without stray fields. Then the dipole field of the screened magnet is equal to

$$B_{ys} = -\frac{\mu_0 jbs}{a+b}\left[1-\frac{a+b}{k(ka+b_s)}\right] = B_y(1-h),\ \ B_{xs}=0$$

The gradient G inside the screened quadrupole lens equals

$$G_s = -\mu_0 j\frac{a-b}{a+b}\left[1-\frac{b}{kb_s}\left(\frac{a+b}{a_s+b_s}\right)^2\right] = G(1-g).$$

Here dimensionless values of h and g show how much the field and gradient of screened magnets differ from the same value of unscreened coils. Evidently, a minimal value of k is $1+s/a$, where s is a distance between centres of the coil pairs. If we choose larger k, then there will be enough a current free space between screening conductors and a main coil to place there force collars that constrain coil motion to negligible levels. The area S of the main dipole coil is equal to

$$S = 2ab\left(\arcsin\frac{s}{2a}+\frac{s}{2a}\sqrt{1-\frac{s^2}{4a^2}}\right) \approx 2bs\left(1-\frac{s^2}{24a^2}\right).$$

The total current in this coil or its ampere-turns Iw are then $Iw = jS$. From this equation and equation (2.3) it follows that the same value for the screening coil Iw_s is

$$Iw_s \approx \frac{Iw}{k}. \tag{2.4}$$

The field strength at the screening coil is very low, and so the critical current density in it is 3—5 times higher than in the main coil. Therefore, the total additional volume of the superconductor in this coil needed for a complete screening is of the order of 10% of the superconductor in the main coil if k is equal to 2.5 and more.

A SC volume needed to screen a quadrupole lens is even lower. The cross section area S_m of its main coil equals

$$S_m = 2ab\left(\arccos\sqrt{\frac{1-e^2}{2-e^2}} - \frac{\pi}{4}\right) \approx \frac{c^2}{2}\sqrt{1-e^2},$$

where $e = ca^{-1}$ is the coil ellipticity. The area of the screening coil is

$$S_s \approx \frac{c^2}{2}\sqrt{1-\frac{e^2}{k^2}}$$

or almost the same as the main coil area. Therefore, Eq.(2.4) in the case of a quadrupole takes the form

$$Iw_s \approx -\frac{Iw}{k^2}.$$

It reflects the fact that a field outside a dipole drops with a distance r as r^{-2}; and outside a quadrupole, as r^{-3}. So the additional SC volume for screening a quadrupole is lower than in the case of a dipole.

3. Dipole with Shielding Coil

The determination of real screened dipole coil geometries is performed in two steps. First, we design the main coil to produce 5T magnetic field with low multipole content. Then kept constant the geometry of main coil we design the shielding coil to screen the field outside the magnet.

Let us consider in cylindrical coordinates r, θ, z, $0 \le \theta \le \frac{\pi}{2}$, M infinitely long straight conductors. Dipole symmetries are implied. Cross section of these conductors has angular positions $\varphi_m - \Delta_m$ and $\varphi_m + \Delta_m$, radial positions R_1^m and R_2^m, $m = 1, ..., M$. Let $r^+ \le \min(R_1^m)$, $r^- \ge \max(R_2^m)$, then for $r < r^+$ magnetic field $B^+(r, \theta)$ inside the aperture due to currents j_m in conductors is described by multipole expansion

$$B^+(r,\theta) = B_y^+ + iB_x^+ = -B_0\left[1 + \sum_{n=1}^{\infty} a_{2n}^+ e^{2n\theta i}\right]$$

$$B_0 = \frac{4\mu_0}{\pi}\sum_{m=1}^{M} j_m (R_2^m - R_1^m)\sin\Delta_m \cos\varphi_m$$

$$a^+_{2n} = \frac{4\mu_0}{\pi}\frac{r^+}{B_0}\frac{1}{4n^2-1}\left(\frac{r}{r^+}\right)^{2n}\sum_{m=1}^{M} j_m\left[\left(\frac{r^+}{R_1^m}\right)^{2n-1} - \left(\frac{r^+}{R_2^m}\right)^{2n-1}\right]\times$$

$$\times \sin(2n+1)\,\Delta_m \cos(2n+1)\,\varphi_m .$$

Magnetic field $B^-(r, \theta)$ outside the magnet is described for $r > r^-$ by

$$B^-(r,\theta) = B_y^- + iB_x^- = \sum_{n=1}^{\infty} a_{2n}^- e^{2n\theta i}$$

$$a^-_{2n} = \frac{4\mu_0}{\pi} r^- \frac{1}{4n^2-1}\left(\frac{r^-}{r}\right)^{2n}\sum_{m=1}^{M} j_m\left[\left(\frac{R_2^m}{r^-}\right)^{2n+1} - \left(\frac{R_1^m}{r^-}\right)^{2n+1}\right]\times$$

$$\times \sin(2n-1)\,\Delta_m \cos(2n-1)\,\varphi_m .$$

The main coil geometries design is considered as an optimization problem with objectives:

- main dipole field of 5T,
- minimization of multipole field — a^+_{2n}, $n = 1, ..., N^+$.

For dipole with shielding coil we add the objective of

- minimization of outside magnetic field — a^-_{2n}, $n = 1, ..., N^-$.

The design variables for this optimization problem are φ_m, Δ_m, j_m, $m = 1, \ldots, M$. Optimization was carried out by NAG Fortran Library routine E04UPF for $M = 6$, $N^+ = N^- = 4$. Geometric parameters of the coil and obtained values of designed variables are presented in Table 1. Table 2 gives the multipole field content for this coil.

Magnetic flux lines and screening effects are depicted in Fig.1. Figure 2 represents total additional volume of superconductor needed for shielding coil (S_a) (in per cent of main coil superconductor volume) in dependence on current density ratio $\frac{j_s}{j_m}$ and on coils radius ratio $\frac{R_s}{R_m}$.

The results above show that SC dipole magnet with low stray fields can be constructed in two-dimensional case. The total additional volume of the superconductor is well described by the formulae

$$S_a = \frac{R_m}{R_s}\frac{j_m}{j_s} S_m ,$$

where S_m is the main coil superconductor volume.

Table 1. Actively screened dipole coil parameters

\m	1	2	3	4	5	6
Δ (rad)	0.514	0.098	0.252	0.063	0.366	0.124
φ (rad)	0.514	1.210	0.252	0.666	0.366	1.030
R_1 (mm)	25	25	35.5	35.5	100	100
R_2 (mm)	35	35	45.5	45.5	101	101
j (A/mm^2)	470	470	470	470	1053	1053

Table 2. Multipole field content for actively screened dipole

2n	$a_{2n}^{+} \times 10^{-4}$, $r = 15$ mm	$a_{2n}^{-} \times 10^{-4}$, $r = 120$ mm
2	1.0066	0.0083
4	0.0006	0.0000
6	−.0002	−.0032
8	0.0000	0.0024
10	0.3920	29.6644
12	−.3395	13.7798

To optimize coil ends, 3D magnetic field calculations were carried out. Obtained early coil geometries are used in calculations.

Magnetic field due to current **j** in coil Ω_c is described by Biot Savart law

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_{\Omega_c} \mathbf{j} \times \nabla \frac{1}{R} \, d\Omega.$$

To calculate **B** we approximate the domain Ω_c by tetrahedral elements with plane faces. Current density **j** is assumed to be a constant vector within the element. Then the evaluation of integral over the domain Ω_c is reduced to summation of integrals over elementary tetrahedrons. Last integrals are evaluated analytically. The magnet coil consist of straight

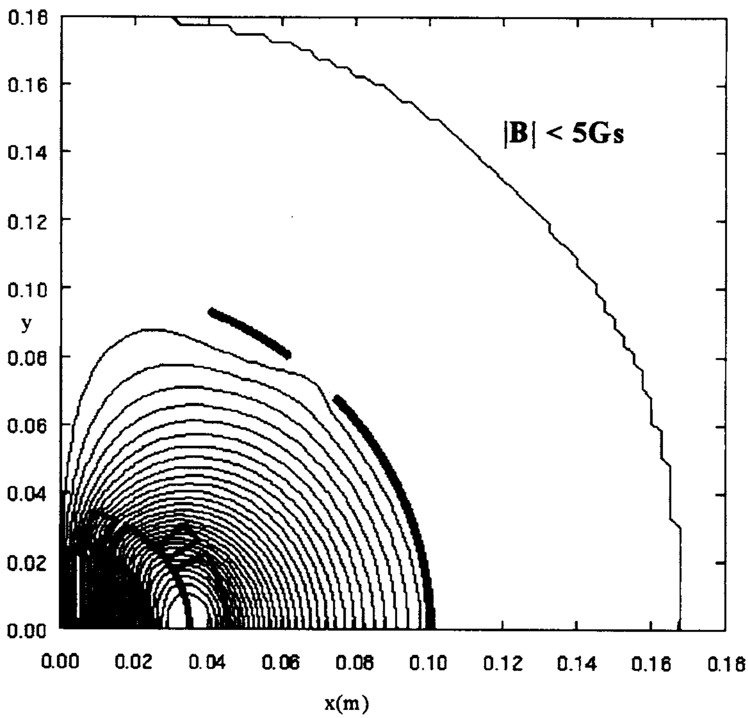

Fig.1. Magnetic field flux lines and screening effects for actively screened dipole

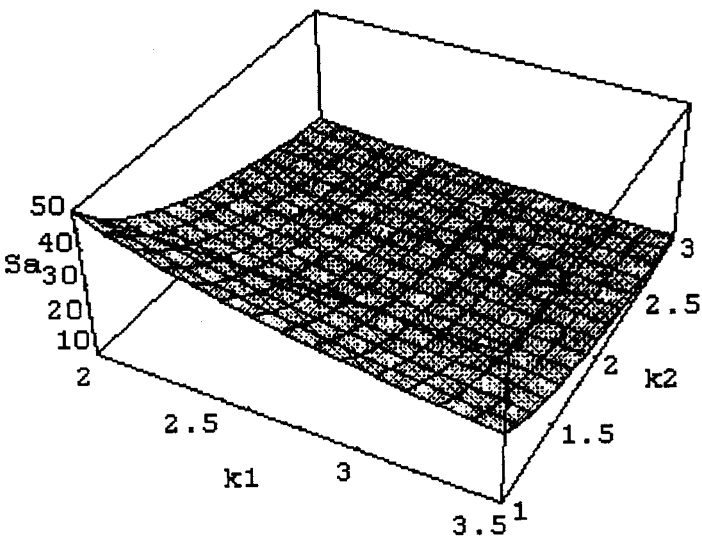

Fig.2. Additional volume of superconductor for actively screened dipole (in per cent of main coil superconductor volume). $k1 = S_s/S_m$, $k2 = j_s/j_m$

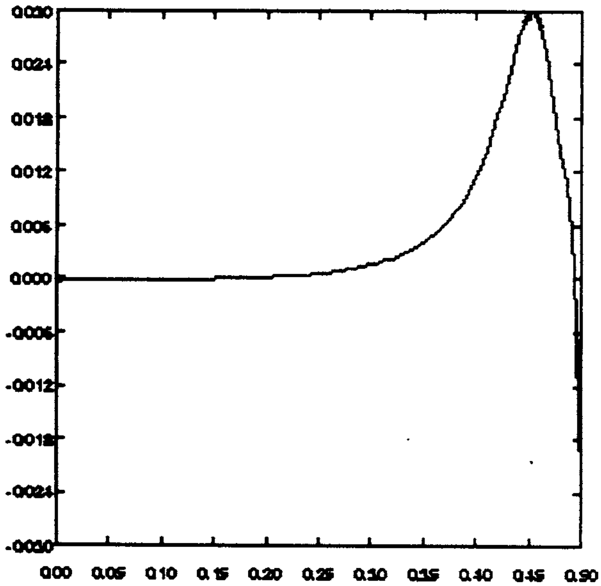

Fig.3. Magnetic field uniformity along the axis of actively screened dipole

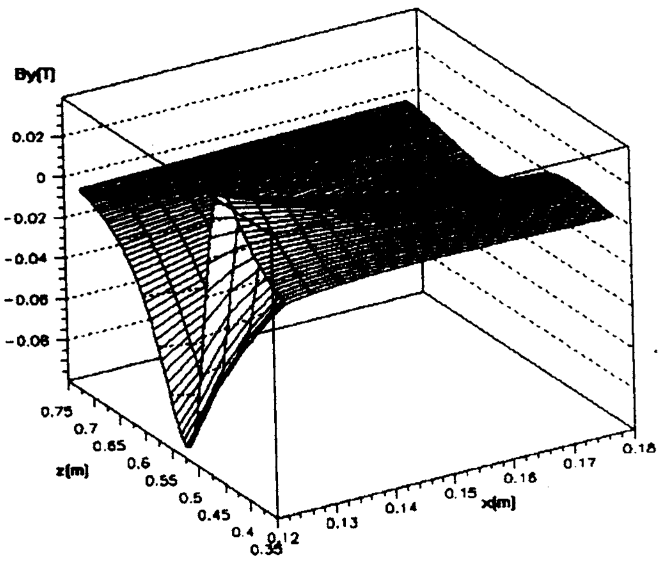

Fig.4. Stray fields in coil end region

Table 3. Actively screened dipole coil ends parameters

\m	1	2	3	4	5	6
d_m (cm)	100.	92.	100.	100.	96.	96.
α_m	1.	1.	1.	1.	0.4	0.4

Table 4. Peak field in actively screened dipole coil blocks

\m	1	2	3	4	5	6
$\|\mathbf{B}\|$ (T)	5.82	5.48	2.62	3.95	1.75	1.73

parts specified by its length d_m, $m = 1, ..., M$ and coil ends specified by its ellipticity α_m, $m = 1, ..., M$. Straight parts and ellipticity of shielding coil blocks are the design variables for coil ends optimization. Two partially contradictory objectives are imposed

- to remain the field uniformity on z axis inside the magnet,
- to screen field outside the magnet.

Field uniformity inside the magnet and magnetic field on median plane outside the magnet are depicted on Figs.3 and 4 for parameters d_m, α_m from Table 3. Table 4 gives the coil blocks peak field.

As it was expected a priori the screening on the magnet end is not so complete than in 2D case but field level is lower than one from the ends of main coil.

4. Conclusions

The results presented here show that wholly screened ironless SC dipoles and quadrupoles may be constructed and successfully used in high energy accelerators. SC magnets of this type are five times lighter per unit length and have two times and more smaller dimensions. It means that the cost of magnet operation could be decreased due to reduced cooling power.

Moreover, these magnets may be used in a collider when two magnets are placed in one cryostat side by side or «two-in-one». The fringing fields of one magnet lower than 5Gs will not disturb the field in the neighbour one. To reach this value it needs to place axes of both magnets at a distance of 4.5—5 radius of the aperture.

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Received on October 8, 1996.